Homework 2 - Solutions

1 Problem 3-7 (Dally and Poulton)

Buses Without Stubs: One can build a bus (multidrop transmission line) in which there are no reflections off the stubs by placing matching networks at each drop across the line, as shown in Figure 3-57. (a)Design the resistive matching network, N, shown in the figure so that a signal transmitted to the network from any of its three terminals is propagated out the other two terminals (possibly attenuated) with no reflections. (b) With the impedance values shown in the figure, 20-Ω bus and 100-Ω stubs, how much energy is lost from the signal traveling down the bus at each stub? (c) How much energy would be lost if both the stubs and the bus were 50-Ω lines?

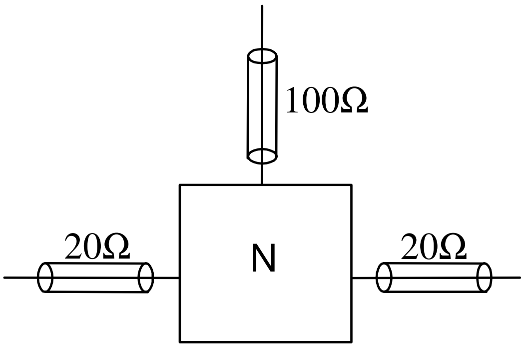
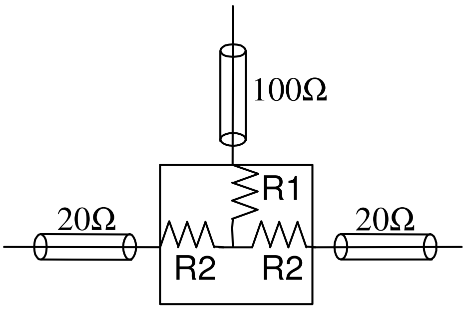


Figure 1: Multidrop Bus

(a) We can make a network of 3 resistors



Thus, we must solve for the following equivalences:

$$20\Omega = R_2 + [(R_2 + 20\Omega)//R_1 + 100\Omega)]$$

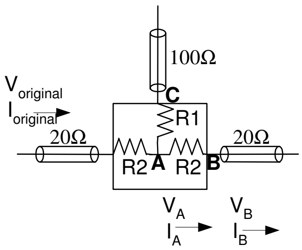
$$100\Omega = R_1 + [(R_2 + 20\Omega)//(R_2 + 20\Omega)]$$

Solving these equations simultaneously, we get:

$$\boxed{R_1 = 89.5\Omega}$$

$$\boxed{R_2 = 1.05\Omega}$$

(b) To solve for the energy lost at each of stubs, we solve for the current and voltage left after 1 stub.



We start by solving for the current and voltage at point A, shown in the figure above. The voltage at point A will be:

$$V_A = \frac{Z_1}{R_2 + Z_1} \times V_{original} = 0.947 \times V_{original}$$

$$, where Z_1 = (R_2 + 20\Omega)//(R_1 + 100\Omega) = 18.9\Omega$$

The current flowing from point A to point B will be:

$$I_A = \frac{R_1 + 100\Omega}{R_1 + 100\Omega + R_2 + 20\Omega} \times I_{original} = 0.9 \times I_{original}$$

The voltage and current at point B will be:

$$V_B = \frac{20\Omega}{20\Omega + R_2} \times V_{original} = 0.95 \times V_A = 0.89 V_{original}$$

$$I_B = I_A$$

Thus, the total power left to pass onto the next stub on the bus is:

$$P_{out} = I_B * V_B = 0.81 * P_{original}$$

So, the power lost at each stub as the signal travels down the line is:

$$\boxed{19\% \text{energy loss at each stub}}$$

We note that 5% of $P_{original}$ continues onto the 100Ω stub at point C, so the actual energy loss we have through our network is only $\boxed{14\%}$. However, the energy loss to the signal continuing down the rest of the bus is, as above stated 81% of $P_{original}$.

(c) Now, if the stubs and the bus were all 50 Ohm, we can go through the exact same procedure to find the resistor values, which turn out to be:

$$\boxed{R_1 = 16.7\Omega}$$

$$\boxed{R_2 = 16.7\Omega}$$

We note that since the network is now symmetric in three ways, we would expect that $R_1 = R_2$ as indeed our results show.

Again, using the same procedure as above, we get:

$$\boxed{75\%\text{energy loss at each stub}}$$

Or, in other words. Only 25% of the original energy, $P_{original}$, is available for the proceeding stubs. Again, as noted above, 25% of the original power also reaches point C in our circuit. So, only $\boxed{50\%}$ of our energy is consumed in our resistive network. However, only 25% of our original signal energy is available to continue on the bus to the rest of the stubs.

We can see the trade-off between the first and second networks. If there are many stubs on the bus, we want the maximum signal power available after passing each stub, as in the first setup. This is especially true if the transmitter stub is far away from the receiver stub. However, if there are few stubs, we want the maximum power to reach the stubs, as in the second setup.

Since a normal bus has a lot of stubs attached to it, the first setup of unequal impedances between stub and bus segment is generally a better idea.

2 Problem 3-11 (Dally and Poulton)

Frequency-dependent Termination:

(1) impedance vs frequency

(a) $Z = R + \frac{1}{Cj\omega} = 50 + \frac{1}{10nFj\omega}$

(b) $Z = j\omega L + R = j\omega 10nH + 50$

(c) $Z = j\omega L + R = j\omega 10nH + 50$

From the equations above, we can plot the graph. (b) and (c) have the same impedance(therefore has the same plot).

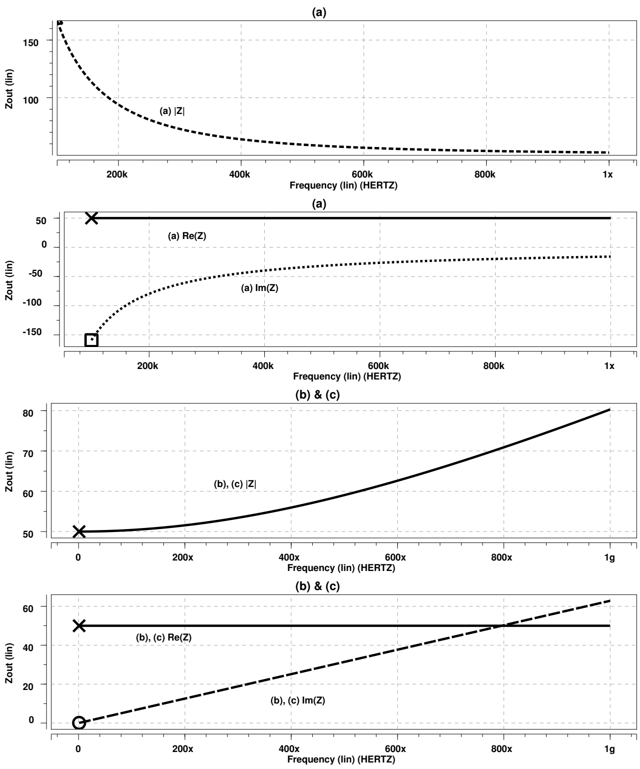
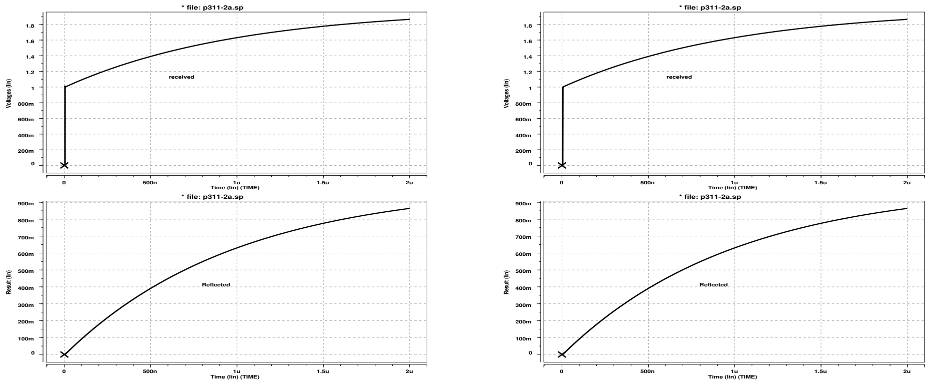


Figure 2: Impedance vs Frequency

(2) waveform received and reflected

(a) At the discontinuity, capacitor act like an short circuit and becomes an open circuit in steady state. At time = 0, there's no reflection since terminal act like matched impedance. But the reflection coefficient goes to 1 in steady state as the capacitor acts like open. Here $\tau = RC = (50 + 50) * 10nF = 1\mu s$, it means that it takes more than $1\mu s$ for the output voltage charged up to final state. Since there's no voltage peaking by the reflection, there's not a big difference between risetime = 100ps and risetime = 1ns.



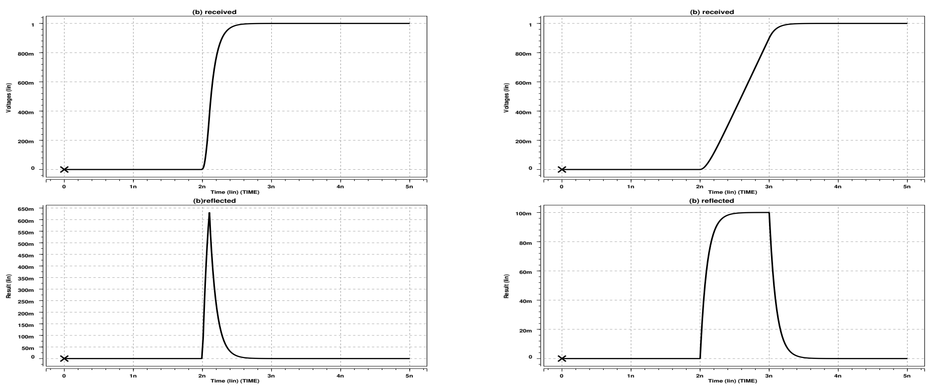
(a) rise time = 100ps (b) rise time = 1ns

Figure 3: Problem 3-11 (a)

(b) Since the inductor acts a an open circuit, the reflected voltage shoots up initially, but eventually goes back down to 0 (when the inductor starts acting as a short circuit). Summing of incident wave and reflection wave is divided to resistor and inductor, and received signal is only on the resistor part. Actual received waveform is

$$\frac{R}{j\omega L+R}(1+\frac{Z_t-Z_0}{Z_t+Z_0})=\frac{R}{j\omega L+R}(\frac{2Z_t}{Z_t+Z_0})$$

So, it start at 0 when incident wave arrives(at discontinuity point V($\omega=\infty$)=0), and goes to 1 at steady state. Time constant for this curve is $\tau=\frac{L}{R}=\frac{10n}{50+50}=0.1ns$ The amplitude of incident wave is 1V(amplitude on the left side of the termination, that is, 2V at the source when it has 50Ohm input resistance with 50Ohm transmission line) with rise time 100ps and 1ns.



(a) rise time = 100ps (b) rise time = 1ns

Figure 4: Problem 3-11 (b)

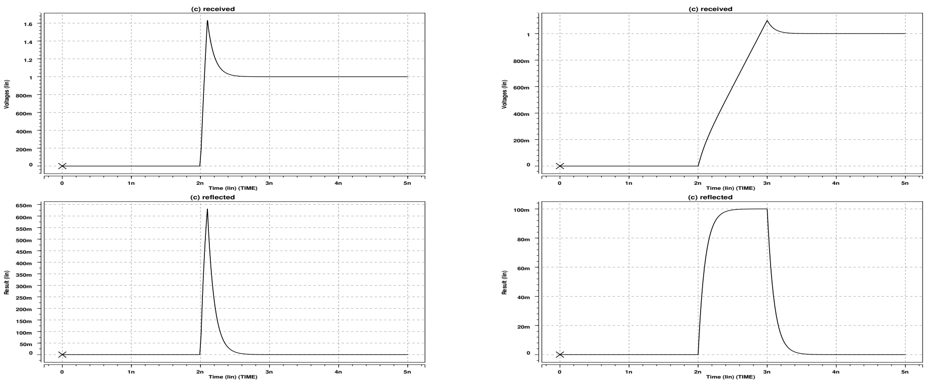
The overshoot of reflected wave is depend on the rise time of the incident wave. So there is a big difference between these two waveforms.

(c) In the beginning, when the inductor acts as an open circuit, it does not let all the voltage through, hence the voltage at the right is less than one. Eventually, the inductor becomes a short circuit and the voltage goes to 1.

The peak overshoot can be calculated as

$$\frac{\Delta V}{V} = (\frac{\tau}{t_r})[1 - exp(-\frac{t_r}{\tau})] = 0.63$$

When rise time = 100ps , $\Delta V = 0.63$,
when rise time = 1ns, $\Delta V = 0.0095$.



(a) rise time = 100ps

(b) rise time = 1ns

Figure 5: Problem 3-11 (c)

3 Problem 3-16 (Dally and Poulton)

Extracting Parasitics: Develop a model circuit composed of ideal transmission lines, inductors, and capacitors that gives the same response as the given waveform. You will want to simulate your model circuit with HSPICE to verify correspondence.

There isn't a clean analytical way of generating the given waveform. We can start with a few equations that give rough estimates of the device values, but because of secondary effects, intermediate reflections, oscillations between inductors and capacitors, etc., eventually we have to resort to the brute force method of trying various parameters until we get a satisfying circuit.

The first step is to determine the arrangement of devices that will exhibit the same behavior as expressed by the waveform. Figure 3-47 of the textbook shows the reflection behavior of a large capacitor and a large inductor. Although we won't see this kind of clean exponential decay following an abrupt change in voltage, it can be observed that a positive bump is probably due to a series inductance, whereas a negative bump is probably due to a parallel capacitor.

There's voltage overshoot at the beginning of TDR waveform, this leads to a inductor at the front of the circuit. The steady state voltage in first plateau means that there are transmission line of impedance Z_1

$$540mV = 1V * \frac{Z_1}{Z_1 + 50} \rightarrow Z_1 = 60$$

And round trip time is around 400ps, so the delay of the transmission line is about 200ps.

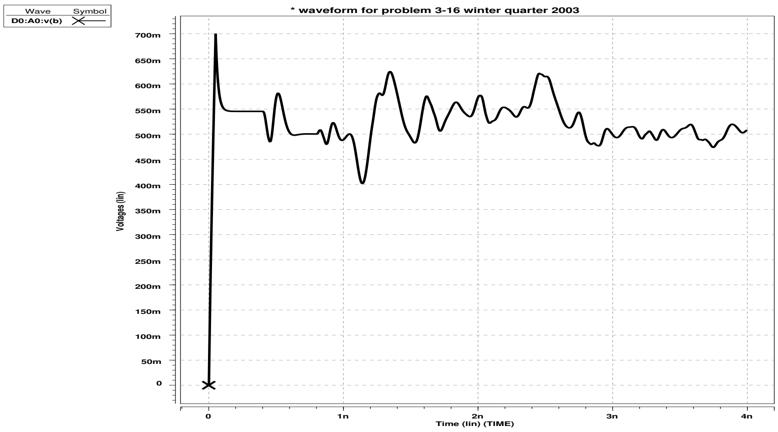


Figure 6: TDR waveform

From the amplitude of overshoot we can calculate τ and inductance

$$\frac{\Delta V}{V} = (\frac{\tau}{t_r})[1 - exp(-\frac{t_r}{\tau})], \; or \; approximately \; \tau = t_r \frac{\Delta V}{V} \; = 15ps \;\; from \; eq3 - 79$$

$$\tau = \frac{L}{R + Z_1}, L = 1.7nH \approx 2nH$$

Then there are downward peak caused by the capacitance C_1 and upward peak caused by inductance L_2. One thing that you should be careful is that the rise time of the wave is changed by the previous inductor. We can calculate the rise time after inductor by using some differential equation but we can also just measure the time constant from the waveform and rise time(about 150ps). There are another transmission line after capacitor and inductor and its impedance is

$$Z(x) = Z_0 \frac{V(2x/v)}{1 - V(2x/v)} = 50 \frac{500mV}{1 - 500mV} = 50$$

So time constant for the capacitance is

$$\tau = t_r \frac{\Delta V}{V} \; = 13.6ps \;\; and \;\; \tau = RC = (50 \parallel 60)C \quad \rightarrow C = 0.5pF$$

From the upward peak in waveform, we can calculate the value of inductance in similar way.

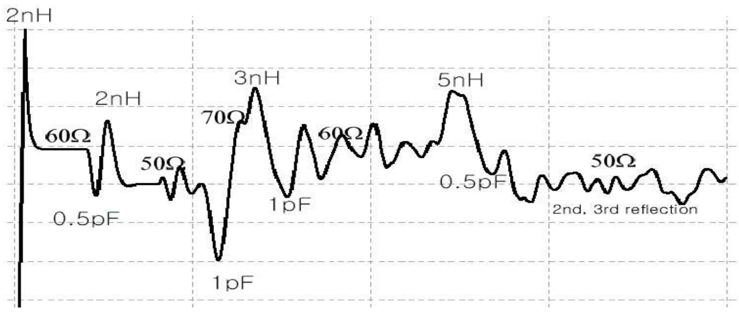


Figure 7: Parasitics extracted

We can calculate other parasitics in the same way up to the end of TDR wavefrom. Above is the value of each element that corresponds to reflection in TDR.

If you don't like this lot of equations to solve this problem, you can also find each value of element by HSPICE simulation. At each reflection, the value of element can be determined by iterative simulation.

The final answer for the parasitic extraction is